Mesoscopic Spin Hall Effect in Multiprobe Semiconductor Bridges

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Abstract. We predict that pure spin current flowing through the transverse leads of quantum-coherent two-dimensional electron gas (2DEG), which is induced by conventional unpolarized charge current driven through the longitudinal leads, can be tuned by the Rashba spin-orbit (SO) interaction and will decay only when disorder is sufficiently strong (but before electrons become localized). Furthermore, the polarization vector of the transported spin is not orthogonal to the plane of 2DEG. Thus, mesoscopic spin Hall current will exhibit markedly different features from recently predicted intrinsic spin Hall effect in macroscopic homogeneous clean semiconductors with SO interaction.

INTRODUCTION

Current efforts in spintronics are to a large extent directed toward gaining control of electron spin in ubiquitous semiconductor structures and exploiting it as a carrier of classical and quantum information [1]. Although dynamics of spin in semiconductor materials is an old subject [2, 3], spintronics has rekindled theoretical interest in such fundamental problems as spin relaxation, spin decoherence, spin transport, and spin injection across various interfaces [4]. For example, it has been known for a long time [3, 5] that SO dependent scattering off impurities deflects spin-up (spin-down) electrons to the right (left), thereby generating a spin current flowing perpendicular to the applied electric field driving the charge current. Such extrinsic spin Hall effect is analogous to traditional charge Hall effect, where Lorentz force due to applied magnetic field separates positive and negative charges along the transverse direction.

Recent theoretically unearthed intrinsic spin Hall effect in hole doped [6] (such as bulk $p$-GaAs or $p$-Ge), or electron [7] doped semiconductors (such as 2DEG in $n$-type heterostructures with broken inversion symmetry) shows that dissipationless transverse spin current could be induced even in clean systems through which conventional unpolarized charge current flows in the longitudinal direction. The correlation between spin orientation and carrier velocity (in the presence of external electric field) induced in this effect essentially requires some type of SO coupling. The induced spin Hall current is pure (i.e., not accompanied by dissipative charge transport) and expected to be much larger than in the case of extrinsic effect. Since it is generated by all-electrical means within a semiconductor environment, it promises to solve one of the major obstacles for spintronics where attempts to inject spin from a metallic ferromagnet into a semiconductor at room temperature are thwarted with low efficiency due to vast difference in their properties [2].
FIGURE 1. The four-probe mesoscopic bridge for the detection of pure spin Hall currents. The central region is 2DEG where electrons are confined within semiconductor heterostructures by an electric field along the $z$-axis that induces the Rashba SO coupling. The four attached leads (semi-infinite in theoretical treatment) are clean, non-magnetic, and without any SO coupling. The unpolarized ($I_s = 0$) charge current ($I_s \neq 0$) through longitudinal leads induces spin Hall current in the transverse leads which act as voltage probes $V_2 = V_3 \neq 0$, $I_2 = I_3 = 0$. In general, the polarization vector of the spin transported by pure ($I_2 = I_2^\uparrow + I_2^\downarrow = 0$) spin current $I_s^2 = \frac{\hbar}{2e}(I_2^\uparrow - I_2^\downarrow)$ will not be orthogonal to the 2DEG.

While theoretical predictions of the spin Hall current properties are based on semiclassical analysis of infinite homogeneous (clean [6, 7] or disordered [9, 10]) semiconductor systems, guiding experimental detection of such effect requires a quantitative prediction for the spin current flowing through the leads attached to a finite-size sample, as exemplified by the bridge in Fig. 1. This is analogous to profound developments in our understanding of quantum Hall effect ensuing from the comparison of macroscopic charge transport in bulk samples to mesoscopic transport through chiral edge states of multi-terminal bridges employed in experiments [13]. For example, within a finite-width strip no charge or spin current can flow across its boundaries. Therefore, non-equilibrium spin accumulation near the transverse edges will generate compensating current in the direction opposite to spin Hall current [12]. Thus, when transverse leads are attached at the boundaries of the wire in Fig. 1, pure ($I_2^\uparrow + I_2^\downarrow = 0$) spin current $I_s^2 = \frac{\hbar}{2e}(I_2^\uparrow - I_2^\downarrow)$ will emerge in the probe 2 of the bridge.

In this letter, we predict that mesoscopic spin Hall current $I_s^2$ in Fig. 1, where central region is quantum-coherent finite-size 2DEG with Rashba interaction, will strongly depend on the strength of the SO coupling. The Rashba electric field along the $z$-axis, that confines the electrons within the $xy$-plane in semiconductor heterostructure, generates an effective momentum-dependent magnetic $B(k)$ field (SO coupling) in the frame of an electron. The spin current $I_s^2$ is also affected by the system size and electron concentration. Moreover, the transported spin will acquire non-zero expectation values along the $x$- and $y$-direction, in addition to purely $z$-axis oriented spin of current flowing along the $y$-axis in bulk samples [6, 7]. Figure 2 summarizes these findings by plotting the spin Hall conductance defined as

$$G_H = \frac{\hbar}{2e} \frac{I_2^\uparrow - I_2^\downarrow}{V_1 - V_4}. \quad (1)$$
FIGURE 2. The dependence of spin Hall conductances $G_{H}^{x}$, $G_{H}^{y}$, $G_{H}^{z}$ in clean four-probe bridges (Fig. 1) on: (a) Fermi energy $[E_F = -2.0t, t_{so}^R = 0.1t]$; (b) strength of the Rashba coupling $[L = 30, E_F = -2.0t]$; and (c) system size $[E_F = -2.0t, t_{so}^R = 0.1t]$; in the inset $E_F = -2.0t, t_{so}^R = 0.1t$.

When disorder (spin-independent scattering off static impurities) is introduced into 2DEG, we evaluate exact $G_{H}$ for non-interacting quasiparticle transport from the quasiballistic to the localized regime, thereby demonstrating in Fig. 3 that spin Hall conductance will gradually diminish as disorder is increased, even for rather large (beyond current experimental capabilities) Rashba coupling.

These features of mesoscopic spin Hall effect can be contrasted to its macroscopic counterpart [6, 7] where electric field $E_x$ driving the charge current along the x-axis through an infinite Rashba spin-split 2DEG induces a pure spin current $j_z^y$ (polarized solely along the z-axis) in the y-direction. The magnitude of the intrinsic spin Hall effect is captured by the spin Hall conductivity $\sigma_H = -j_z^y/E_x$. Surprisingly enough, this quantity, for both 3D hole-doped [6] and 2D electron-doped [7, 9] semiconductor
candidate systems, is predicted to acquire a universal value $\sigma_H = e / 8\pi$ which apparently does not depend on the strength of respective SO interaction.

**MULTIPROBE SPIN QUANTUM TRANSPORT**

The mesoscopic experiments on quantum Hall bridges in the early 1980s have posed a challenge for theoretical interpretation of multi-terminal transport measurements. By viewing the current and voltage probes on equal footing, Büttiker has provided an elegant multiprobe formula [14], $I_p = \sum_q G_{pq}(V_p - V_q)$, which relates current in probe $p$ to voltages $V_q$ in all probes attached to the sample via conductance coefficients $G_{pq}$. To study the spin-resolved currents of individual spin species $\uparrow, \downarrow$ we imagine that each non-magnetic lead in Fig. 1 consists of the two leads allowing only one spin species to propagate. Upon replacement $I_p \rightarrow I_p^\alpha$ and $G_{pq} \rightarrow G_{pq}^{\alpha\alpha'}$ ($\alpha = \uparrow, \downarrow$), this viewpoint allows us to transform the standard formulas for unpolarized charge transport into the multiprobe ones for spin resolved currents, thereby obtaining the linear response relation for spin current $I_p^\alpha = \frac{\hbar}{2e}(I_p^\uparrow - I_p^\downarrow)$ in the lead $p$

$$I_p^\alpha = \frac{\hbar}{2e} \sum_q [(G_{qp}^{\uparrow\uparrow} + G_{qp}^{\downarrow\downarrow} - G_{qp}^{\uparrow\downarrow} - G_{qp}^{\downarrow\uparrow})V_p$$

$$- (G_{pq}^{\uparrow\uparrow} + G_{pq}^{\downarrow\downarrow} - G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow})V_q]. (2)$$

Since the total charge currents $I_p = I_p^\uparrow + I_p^\downarrow$ depend only on voltage difference between the leads in Fig. 1, we set one of them to zero (e.g., $V_4 = 0$). By inverting the multiprobe equations for charge transport to get the voltages, one obtains ratios $V_2/V_1$ and $V_3/V_1$, where $V_2/V_1 = V_3/V_1 = 0.5$ in ballistic bridges due to the symmetry relation $G_{pq} = G_{qp}$ satisfied by the conductance coefficients in the absence of magnetic field [14]. Finally, by solving Eqs. (2) for $I_2^\alpha$ we obtain an explicit expression for the spin Hall conductance defined in Eq. (1)

$$G_H = \frac{\hbar}{2e} \left[ (G_{12}^{\text{out}} + G_{32}^{\text{out}} + G_{42}^{\text{out}}) \frac{V_2}{V_1} - \frac{G_{23}^{\text{in}} V_3}{V_1} - \frac{G_{21}^{\text{in}}}{V_1} \right]. (3)$$

Here we simplify the notation by using the following labels: $G_{pq}^{\text{in}} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\downarrow\downarrow} - G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow}$ and $G_{pq}^{\text{out}} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\downarrow\downarrow} - G_{pq}^{\uparrow\downarrow} - G_{pq}^{\downarrow\uparrow}$ (the usual charge conductance coefficients [14, 15] are $G_{pq} = G_{pq}^{\uparrow\uparrow} + G_{pq}^{\downarrow\downarrow} + G_{pq}^{\uparrow\downarrow} + G_{pq}^{\downarrow\uparrow}$).

At zero temperature, spin-resolved conductance coefficients $G_{pq}^{\alpha\alpha'} = e^2 \sum_i |t_{ij}^{pq}|^2$ are proportional to the probability that spin-$\alpha'$ electron incident in lead $q$ will be transmitted to lead $p$ as spin-$\alpha$ electron. The quantum-mechanical probability amplitude for this processes is given by the matrix elements of the Landauer transmission matrix $t^{pq}$. In our bridge $G_{pq}^{\alpha\alpha'} = G_{qp}^{\alpha' - \alpha}$ since SO couplings do not break time reversal invariance.

For single particle propagation through a finite-size sample, the transmission matrices between different leads can be evaluated in a numerically exact fashion for arbitrary disorder and Rashba SO coupling strength using the real⊗spin-space Green functions [15].
The retarded Green function \( G^r = [E - \hat{H} - \hat{\Sigma}^r]^{-1} \) (\( \hat{\Sigma}^r \) is the sum of self-energies introduced by the four leads) is obtained by inverting the Rashba Hamiltonian which models the 2DEG region in Fig. 1

\[
\hat{H} = \sum_m \varepsilon_m |m\rangle \langle m| - t \sum_{|m,m'|} |m\rangle \langle m'| + \frac{\alpha \hbar}{2a^2 t} (\hat{\nu}_y \otimes \hat{\sigma}_x - \hat{\nu}_x \otimes \hat{\sigma}_y)
\]

The Hamiltonian, represented in a local orbital basis, is defined on the \( L \times L \) lattice where \( t \) is the nearest-neighbor hopping integral between \( s \)-orbitals \( \langle r|m\rangle = \psi(r-m) \) on adjacent atoms located at sites \( m = (m_x, m_y) \) of the lattice. The hopping sets the unit of energy \( t = 1 \). Here \( \otimes \) stand for the tensor product of operators (i.e., Kronecker product of matrix representations). Since velocity operator in the tight-binding representation is \( \langle m|\hat{v}_x|m'\rangle = \frac{i}{\hbar} t (m_x - m'_x) \), the energy scale of the Rashba SO term in Eq. (4) is set by the "SO hopping parameter" \( t_{so}^R = \alpha/2a \). For perfectly clean 2DEG the on-site potential energy is \( \varepsilon_m = 0 \), while disorder will be simulated using standard random variable \( \varepsilon_m \in [-W/2, W/2] \) that models short-range isotropic scattering off spin-independent impurities.

**SPIN HALL CURRENT IN BALLISTIC 4-PROBE BRIDGES**

In general, the spin Hall conductance \( G^H(E_F, t_{so}^R, L, W) \) is determined by the Fermi energy \( E_F \) (note that \( \sigma^H \) is not a Fermi surface property since it can be expressed as an integral over all states below \( E_F \) [6]), system size \( L \), SO coupling \( t_{so}^R \), and disorder strength \( W \). Moreover, it can be expected that detection of mesoscopic spin Hall current by using ferromagnetic voltage probe, whose magnetization sets the spin quantization axis, will yield spin expectation values to be non-zero for all three axes. To investigate this possibility we, therefore, study three different spin Hall conductances \( G^H_x, G^H_y, G^H_z \), where spin quantization axis for \( \uparrow, \downarrow \) in Eq. (1) is chosen to be \( x-, y-, \) or \( z- \) axis, respectively. Indeed, Figure 2 demonstrates that \( G^H_x, G^H_y, G^H_z \) for 2DEG with no impurities turn out to be non-zero, where \( G^H_z \) is the largest one. They explicitly depend on the value of the SO coupling and the system size. The oscillatory behavior of \( G^H(E_F) \), which becomes more conspicuous in smaller systems, is a consequence of fully quantum-coherent transport through the 2DEG of the mesoscopic bridge in Fig. 1. Considerable interest for exploiting the Rashba SO coupling in semiconductor spintronic application stems from the possibility to tune its strength via gate electrode [16], thereby manipulating spin solely by electrical means. Despite recent experimental efforts [16], the largest achieved value is \( t_{so}^R \sim 0.01 t \).

On the other hand, conductivity \( \sigma^H = e/8\pi \) characterizing macroscopic spin Hall current is independent of both \( \alpha \) and the density of conduction electrons (on the proviso that both bands split by the SO coupling are occupied) [7]. The independence of \( \sigma^H \) on the SO coupling is rather unphysical results which has been carefully reexamined by taking into account finite momentum relaxation time in realistic semiconductors which always contain some impurities [9]. Our treatment of purely ballistic mesoscopic system...
FIGURE 3. The effect of disorder on spin Hall conductances \( \langle G_{yH} \rangle \), \( \langle G_{zH} \rangle \), \( \langle G_{xH} \rangle \) (at \( E_F = -2.0 \)) in the four-probe mesoscopic bridges (Fig. 1 where 2DEG is modeled on the 30 × 30 lattice) with different strengths of the Rashba coupling. In the shaded range of \( W \) both the disorder-averaged \( \langle G \rangle \) and the typical \( e^{\langle \ln G \rangle} \) two-probe charge conductance diminish below \( \simeq 0.1 \left( 2e^2/h \right) \) due to strong localization effects.

does not face any technical difficulties in locating the lower, as well as the upper, limit on the SO coupling \( 10^{-3} t \lesssim t_{so}^R \lesssim 5.0 t \) [see panel (b) in Fig. 2] that is responsible for non-negligible spin Hall conductance \( G_{H} \gtrsim 10^{-2} (e/8\pi) \).

Naively, one would expect that \( G_{H} \equiv \sigma_{H} \) since it is tempting to identify \( j_{z} = I_{2}/L \) and \( E_x = V_1/L. \) However, the plausible spin current operator \( \hat{j}_k^z = \hbar/4 (\hat{\sigma}_z \hat{v}_k + \hat{v}_k \hat{\sigma}_z) \), employed in different computational schemes for the bulk spin Hall conductivity [6, 7, 9–11, 18], lacks rigorous theoretical justification [6]. That is, its relation to real transport of spins remains unclear [11] because of the fact that spin is not conserved quantity in the presence of SO interactions. Contrary to the confusion surrounding usage of \( \hat{j}_k^z \), the pure spin current \( I_{2}^\uparrow - I_{2}^\downarrow \) defined and evaluated within the asymptotic region of the leads (where \( t_{so}^R \equiv 0 \)) has transparent physical interpretation: If all spin-\( \uparrow \) electrons move in one direction, while an equal number of spin-\( \downarrow \) move in the opposite direction, the net charge current vanishes while spin current can be non-zero. Such currents have been created and detected in recent pump-probe experiments [17]. The asymmetry of \( G_H(E_F) \) with respect to \( G(E_F = 0) \equiv 0 \) at the half-filled band in Fig. 2(a) is a general property of spin currents defined in this fashion—transport above half-filling can be interpreted as the propagation of positively charged holes which move in the opposite direction, and have opposite spin, to that of electrons.
SPIN HALL CURRENT IN DISORDERED 4-PROBE BRIDGES

The major difference between the "old" extrinsic [3, 5] and the "new" intrinsic spin Hall effect is that the former vanishes in the clean limit, while the later persists even when no skew-scattering at impurities takes place [6, 7]. In fact, resilience of the intrinsic effect to disorder has become a major issue in current debates over the observability of spin Hall current in realistic samples of 2DEG with the Rashba SO interaction [9–11, 18] or in hole-doped bulk 3D semiconductors [8, 9]. In particular, two sets of contradicting results have emerged in recent transport calculations for infinite homogeneous 2DEG: (i) spin Hall current is suppressed even for vanishingly small disorder [11]; (ii) $\sigma_H \neq 0$ (including even the possibility of $\sigma_H > e/8\pi$ [10]) when SO splitting energy is larger than the disorder broadening of the energy levels [9, 18].

Here we shed light on the effect of disorder on spin Hall current in mesoscopic systems by introducing static random potential into the 2DEG region of Fig. 1 and studying the disorder-averaged conductances $\langle G^x_H \rangle$, $\langle G^y_H \rangle$, $\langle G^z_H \rangle$ in the crossover from the quasiballistic to the localized transport regime. The inclusion of all transport regimes is made possible by employing exact single-particle spin-dependent Green function [15] which encompasses all quantum-interference effects at arbitrary $W$ (i.e., in both weak [11, 18] and strong disorder) and $t_{so}^R$, rather than treating only the lowest order (semiclassical) effects of the disorder [9–11, 18]. Figure 3 demonstrates that spin Hall conductances are unaffected by weak disorder (similarly to recent numerical findings on the behavior of $\sigma_H$ [18]), but will gradually diminish to zero when $W \gg t_{so}^R$ within the diffusive transport regime.

The two-probe (charge) conductance of sufficiently large $L \gg \xi$ 2DEG with impurities will exhibit exponential decay $G \sim e^{-L/\xi}$. The two exceptions are quantum Hall 2DEG (where delocalized states exist in the center of a Landau level) [13] and 2D system where sufficiently strong SO coupling can induce a metallic phase $\xi \to \infty$ at weak disorder [19]. We delineate in Fig. 3 the boundary [19] of localization-delocalization transition to demonstrate that $G_H$ will vanish before 2DEG is pushed into the realm of strong localization (where $G_H \to 0$ can be trivially expected).

CONCLUSIONS

Our analysis suggests that pure spin current, flowing in the transverse direction in response to longitudinal unpolarized quantum-coherent transport of charge through experimentally relevant system (Hall bar consisting of a finite-size 2DEG attached to four current and voltage probes), should display the following observable features: (a) it is non-zero within a finite interval of the Rashba SO coupling values; (b) its spin has expectation values in all three direction, where $G^y_H$ is the largest of the three spin conductances; (c) $G^x_H$ and $G^y_H$ increase with the system size, while $G^z_H$ has much weaker size dependence at large SO coupling; and (d) it gradually decays with increasing disorder, reaching zero before the onset of strongly localized phase, even in systems with sub-
stantial SO coupling. The spin Hall effect dichotomy—mesoscopic (in inhomogeneous phase-coherent structures) versus macroscopic [6, 7, 9] (a semiclassical phenomenon driven by external electric field in infinite semiconductors)—originates from the sensitivity of the dynamics of transported spin in SO coupled semiconductors to finite-size, interface, and quantum-coherent effects [15, 20].

ACKNOWLEDGMENTS

This research has been supported in part by the American Chemical Society grant PRF-41331-G10.

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